Eur. Phys. J. C 24, 281–285 (2002)
Digital Object Identifier (DOI) 10.1007/s100520200927

THE EUROPEAN
PHYSICAL JOURNAL C

Proton polarizability contribution to hydrogen hyperfine splitting

R.N. Faustov[1,a], A.P. Martynenko[2,b]

[1] Russian Academy of Sciences, Scientific Council for Cybernetics, Vavilov Street 40, Moscow 117333, Russia
[2] Samara State University, Acad. Pavlov 1, Samara 443011, Russia

Received: 10 July 2000 /
Published online: 12 April 2002 –

Abstract. The contribution of proton polarizability to hydrogen hyperfine splitting is evaluated on the basis of modern experimental and theoretical results on the proton polarized structure functions. The value of this correction is equal to 1.4 ppm.

The investigation of the hyperfine splitting (HFS) of the hydrogen atom ground state has been considered for many years as an important test of quantum electrodynamics. The experimental value of the hydrogen hyperfine splitting was obtained with very high accuracy [1]:

$$\Delta E^{\rm exp}_{\rm HFS} = 1420405.7517667(9)\,{\rm kHz}. \quad (1)$$

The corresponding theoretical expression of the hydrogen hyperfine splitting may now be written in the form [2]

$$\Delta E^{\rm th}_{\rm HFS} = E_F(1+\delta^{\rm QED}+\delta^S+\delta^P),$$
$$E_F = \frac{8}{3}\alpha^4\frac{\mu_p m_p^2 m_e^2}{(m_p+m_e)^3}, \quad (2)$$

where μ_p is the proton's magnetic moment, and m_e, m_p are the masses of the electron and proton. The calculation of different corrections to E_F has a long history. The present status in the theory of hydrogenic atoms was presented in detail in [3]. $\delta^{\rm QED}$ denotes the contribution of higher-order quantum-electrodynamical effects. The corrections δ^S and δ^P take into account the influence of the strong interaction. δ^S describes the effects of the proton's finite size and recoil contribution. δ^P is the correction due to the proton polarizability. The basic uncertainty of the theoretical result (2) is related with this term.

The main contribution to δ^P is determined by the two-photon diagrams shown in Fig. 1. The corresponding amplitudes of virtual Compton scattering on the proton can be expressed through the nucleon polarized structure functions $G_1(\nu,Q^2)$ and $G_2(\nu,Q^2)$. The inelastic contribution of the diagrams of Fig. 1a,b may be presented in the form [3,4,26,6–8]

$$\Delta E^P_{\rm HFS} = \frac{Z\alpha m_e}{2\pi m_p(1+\kappa)}E_F(\Delta_1+\Delta_2)$$
$$= (\delta^P_1+\delta^P_2)E_F = \delta^P E_F, \quad (3)$$

[a] e-mail: faustov@theory.npi.msu.su
[b] e-mail: mart@ssu.samara.ru

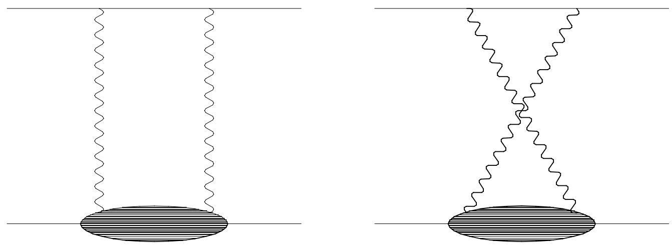

Fig. 1a.b. Feynman diagrams for the proton's polarizability correction to the hydrogen HFS

$$\Delta_1 = \int_0^\infty \frac{{\rm d}Q^2}{Q^2}\left\{\frac{9}{4}F_2^2(Q^2) - 4m_p^3\int_{\nu_{\rm th}}^\infty \frac{{\rm d}\nu}{\nu}\beta_1\left(\frac{\nu^2}{Q^2}\right)G_1(\nu,Q^2)\right\}, \quad (4)$$

$$\Delta_2 = -12m_p^2\int_0^\infty \frac{{\rm d}Q^2}{Q^2}\int_{\nu_{\rm th}}^\infty {\rm d}\nu\beta_2\left(\frac{\nu^2}{Q^2}\right)G_2(\nu,Q^2), \quad (5)$$

where $\nu_{\rm th}$ determines the pion–nucleon threshold:

$$\nu_{\rm th} = m_\pi + \frac{m_\pi^2+Q^2}{2m_p}, \quad (6)$$

and the functions $\beta_{1,2}$ have the form

$$\beta_1(\theta) = 3\theta - 2\theta^2 - 2(2-\theta)\sqrt{\theta(\theta+1)}, \quad (7)$$
$$\beta_2(\theta) = 1+2\theta-2\sqrt{\theta(\theta+1)}, \quad \theta = \nu^2/Q^2. \quad (8)$$

$F_2(Q^2)$ is the Pauli form factor of the proton, κ is the proton anomalous magnetic moment: $\kappa = 1.792847386(63)$ [9]. During many years there were not enough experimental data and there was not enough theoretical information about the proton's spin-dependent structure functions, so the previous study of the contribution $\Delta E^P_{\rm HFS}$ contains only an estimation of the proton's polarizability effects: $\delta^P \sim$ 1–2 ppm or the calculation of the main resonance contributions [6–8,10]. The theoretical bound for the proton's polarizability contribution is $|\delta^P| \leq 4$ ppm. As noted in [3], the problem of the proton's polarizability contribution requires a new investigation which takes into account more recent experimental data on the spin structure of the nucleon.

The polarized structure functions $g_1(\nu, Q^2)$ and $g_2(\nu, Q^2)$ enter in the antisymmetric part of the hadronic tensor $W_{\mu\nu}$, describing lepton–nucleon deep inelastic scattering [11]:

$$W_{\mu\nu} = W_{\mu\nu}^{[S]} + W_{\mu\nu}^{[A]}, \tag{9}$$

$$W_{\mu\nu}^{[S]} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) W_1(\nu, Q^2) + \left(P_\mu - \frac{P\cdot q}{q^2} q_\mu\right)\left(P_\nu - \frac{P\cdot q}{q^2} q_\nu\right)\frac{W_2(\nu, Q^2)}{m_p^2}, \tag{10}$$

$$W_{\mu\nu}^{[A]} = \epsilon_{\mu\nu\alpha\beta} q^\alpha \left\{ S^\beta \frac{g_1(\nu, Q^2)}{P\cdot q} + [(P\cdot q)S^\beta - (S\cdot q)P^\beta]\frac{g_2(\nu, Q^2)}{(P\cdot q)^2}\right\}, \tag{11}$$

where $g_1(\nu, Q^2) = m_p^2 \nu G_1(\nu, Q^2)$ and $g_2(\nu, Q^2) = m_p \nu^2 G_2(\nu, Q^2)$; $q^2 = -Q^2$ is the square of the four-momentum transfer. The invariant quantity $P\cdot q$ is related to the energy transfer ν in the proton's rest frame: $P\cdot q = m_p\nu$. The invariant mass of the electroproduced hadronic system, W, is then $W^2 = m_p^2 + 2m_p\nu - Q^2$.

The proton spin structure functions can be measured in the inelastic scattering of polarized electrons on polarized protons. Recent improvements in polarized lepton beams and targets have made it possible to make increasingly accurate measurements of the nucleon's polarized structure functions $g_{1,2}$ in experiments at SLAC, CERN and DESY [12–19]. The spin-dependent structure functions may be expressed in terms of virtual photon-absorption cross sections [11]:

$$g_1(\nu, Q^2) = \frac{m_p \cdot K}{8\pi^2\alpha(1+Q^2/\nu^2)} \times \left[\sigma_{1/2}(\nu, Q^2) - \sigma_{3/2}(\nu, Q^2) + \frac{2\sqrt{Q^2}}{\nu}\sigma_{\mathrm{TL}}(\nu, Q^2)\right], \tag{12}$$

$$g_2(\nu, Q^2) = \frac{m_p \cdot K}{8\pi^2\alpha(1+Q^2/\nu^2)} \times \left[-\sigma_{1/2}(\nu, Q^2) + \sigma_{3/2}(\nu, Q^2) + \frac{2\nu}{\sqrt{Q^2}}\sigma_{\mathrm{TL}}(\nu, Q^2)\right], \tag{13}$$

where $K = \nu - (Q^2/(2m_p))$ is the Hand kinematical flux factor for virtual photons, and $\sigma_{1/2}$, $\sigma_{3/2}$ are the virtual photoabsorption transverse cross sections for a total helicity for photon and nucleon of 1/2 and 3/2 respectively, σ_{TL} is the interference term between the transverse and longitudinal photon–nucleon amplitudes. In this work we calculate the contribution $\Delta E^P_{\mathrm{HFS}}$ on the basis of modern experimental data on the structure functions $g_{1,2}(\nu, Q^2)$ and theoretical predictions on the cross sections $\sigma_{1/2,3/2,\mathrm{TL}}$.

To obtain the correction (3) at the resonance region ($W^2 \leq 4\,\mathrm{GeV}^2$) we use the Breit–Wigner parameterization for the photoabsorption cross sections in (12) and (13), suggested in [20–26]. There are many baryon resonances that give a contribution to the photon-absorption cross sections. We take into account only five important resonances: $P_{33}(1232)$, $S_{11}(1535)$, $D_{13}(1520)$, $P_{11}(1440)$ and $F_{15}(1680)$. Considering the one-pion decay channel of the resonances, the absorption cross sections $\sigma_{1/2}$ and $\sigma_{3/2}$ may be written as follows [23,27]:

$$\sigma_{1/2,3/2} = \left(\frac{k_R}{k}\right)^2 \frac{W^2 \Gamma_\gamma \Gamma_{R\to N\pi}}{(W^2 - M_R^2)^2 + W^2\Gamma_{\mathrm{tot}}^2}\frac{4m_p}{M_R\Gamma_R} \times |A_{1/2,3/2}|^2, \tag{14}$$

where $A_{1/2,3/2}$ are the transverse electromagnetic helicity amplitudes, and we have

$$\Gamma_\gamma = \Gamma_R\left(\frac{k}{k_R}\right)^{j_1}\left(\frac{k_R^2 + X^2}{k^2+X^2}\right)^{j_2}, \quad X = 0.3\,\mathrm{GeV}. \tag{15}$$

The resonance parameters Γ_R, M_R, j_1, j_2, Γ_{tot} were taken from [9,28]. In accordance with [22,24,28] the parameterization of the one-pion decay width is

$$\Gamma_{R\to N\pi}(q) = \Gamma_R \frac{M_R}{m_p}\left(\frac{q}{q_R}\right)^3\left(\frac{q_R^2 + C^2}{q^2 + C^2}\right)^2, \quad C = 0.3\,\mathrm{GeV} \tag{16}$$

for $P_{33}(1232)$ and

$$\Gamma_{R\to N\pi}(q) = \Gamma_R\left(\frac{q}{q_R}\right)^{2l+1}\left(\frac{q_R^2+\delta^2}{q^2+\delta^2}\right)^{l+1} \tag{17}$$

for $D_{13}(1520)$, $P_{11}(1440)$ and $F_{15}(1680)$. l is the pion angular momentum and $\delta^2 = (M_R - m_p - m_\pi)^2 + \Gamma_R^2/4$. Here k and q are the photon and pion 3-momentum in the cms for a given center of mass energy W, k_R and q_R are taken at the pole of the resonance. In the case of $S_{11}(1535)$ we take into account the πN and ηN decay modes [24,28]

$$\Gamma_{R\to\pi,\eta} = \frac{q_{\pi,\eta}}{q} b_{\pi,\eta}\Gamma_R\frac{q^2_{\pi\eta} + C^2_{\pi,\eta}}{q^2 + C^2_{\pi,\eta}}, \tag{18}$$

where $b_{\pi,\eta}$ are the π (η) branching ratio.

The cross section σ_{TL} is determined by an expression similar to (14), containing the product $(S^*_{1/2}\cdot A_{1/2} + A^*_{1/2}S_{1/2})$ [12]. The calculation of the helicity amplitudes $A_{1/2}$, $A_{3/2}$ and the longitudinal amplitude $S_{1/2}$, as functions of Q^2, was done on the basis of the constituent quark model in [30–34]. In the real photon limit, $Q^2 = 0$, we take corresponding resonance amplitudes from [9]. For the Δ isobar amplitudes $A_{1/2}(Q^2)$, $A_{3/2}(Q^2)$ we used the relations obtained in [35]. Helicity amplitudes of the other resonances were taken from [31–34]. We have considered the Roper resonance $P_{11}(1440)$ as an ordinary qqq state. As follows from the predictions of the quark model, the helicity amplitudes, which may be suppressed at $Q^2 = 0$,

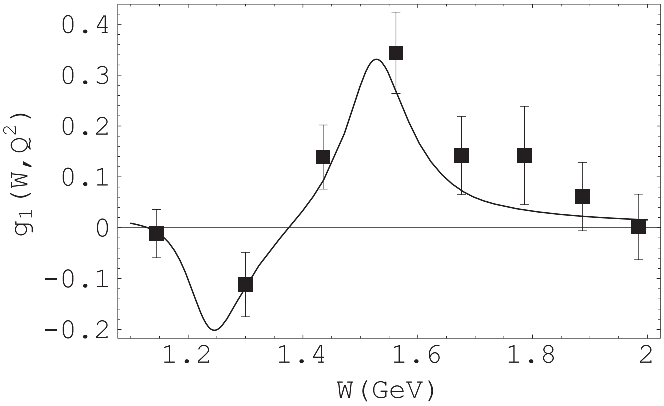

Fig. 2. Proton structure function $g_1(W,Q^2)$ for $Q^2 = 0.5$ in the resonance region. Experimental points correspond to [12]

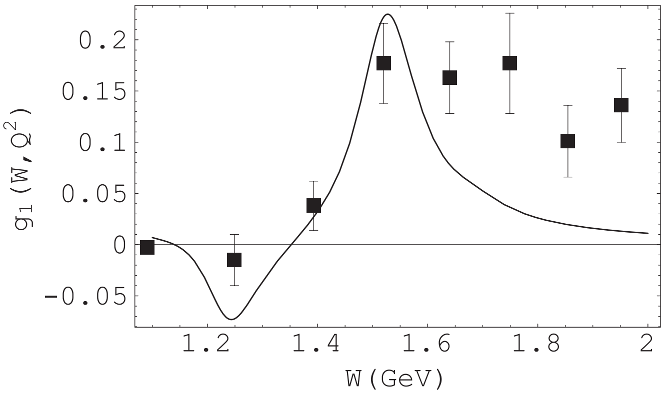

Fig. 3. Proton structure function $g_1(W,Q^2)$ for $Q^2 = 1.2$ in the resonance region. Experimental points correspond to [12]

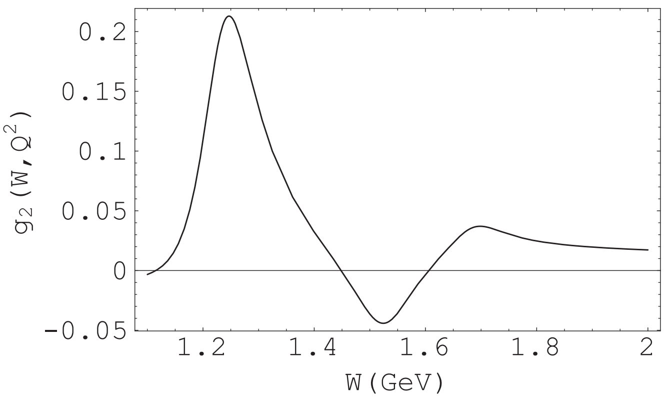

Fig. 4. Proton structure function $g_2(W,Q^2)$ for $Q^2 = 0.5$ in the resonance region

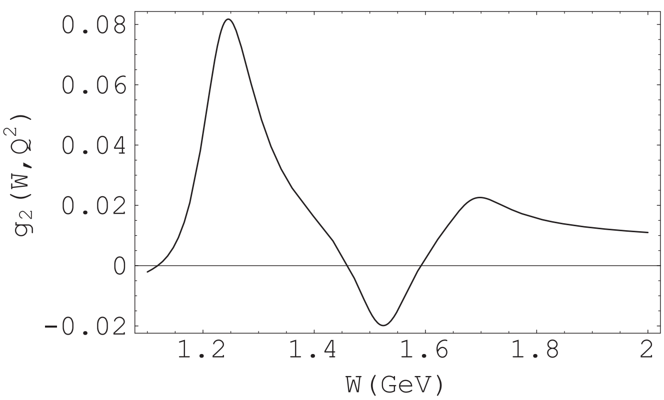

Fig. 5. Proton structure function $g_2(W,Q^2)$ for $Q^2 = 1.2$ in the resonance region

become dominant very rapidly with Q^2. This may be seen on Figs. 2–5, where we have also shown experimental data of the E143 collaboration at two fixed momentum transfer points: $Q^2 \approx 0.5\,\mathrm{GeV}^2$ and $Q^2 \approx 1.2\,\mathrm{GeV}^2$. Our results for the structure function $g_1(\nu,Q^2)$ on Figs. 2 and 3, which are in qualitative agreement with [27] and experimental data, show that the Breit–Wigner five resonance parameterization of the photon cross sections and the constituent quark model results give a good description of the proton's polarized structure functions at the resonance region. The existing difference of this model for $g_{1,2}(\nu,Q^2)$ and the experimental data, which is in particular seen in Fig. 3, demands further improvement in the construction of spin-dependent structure functions. This may be done considering the contributions of other baryonic resonances in the large W range: $S_{31}(1620)$, $F_{37}(1950)$, $D_{33}(1700)$, $P_{13}(1720)$ and $F_{35}(1905)$, and accounting different decay modes of such states [27]. The sum rule of Gerasimov–Drell–Hern [36]

$$-\frac{\kappa^2}{4m_p^2} = \frac{1}{8\pi^2\alpha}\int_{\nu_{\mathrm{th}}}^{\infty}\frac{\mathrm{d}\nu}{\nu}[\sigma_{1/2}(\nu,0)-\sigma_{3/2}(\nu,0)] \qquad (19)$$

is valid with high accuracy [27]. The second part of (4) gives in particular a large negative contribution to the correction δ_1^P in the range of small Q^2, where the contribution of the Δ isobar is dominant. With increasing Q^2 its value decreases and the total correction δ_1^P has a positive sign.

Our calculation of the contribution $\Delta E^P_{\mathrm{HFS}}$ in the DIS region ($W^2 \geq 4\,\mathrm{GeV}^2$) is based on recent experimental data [12–19]. All of the data, including the SMC data at $Q^2 \leq 1\,\mathrm{GeV}^2$, were fit to the parameterization

$$g_1(x,Q^2) = a_1x^{a_2}(1+a_3x+a_4x^2)[1+a_5f(Q^2)]F_1(x,Q^2), \qquad (20)$$

where $x = Q^2/2m_p\nu$ is the Bjorken scaling variable, $F_1 = W_1m_p$. The coefficients of the fits and different models for the form of the Q^2 dependence may be found in [12,19]. In Figs. 6 and 7 the experimental data and parameterization in the form (20) for the ratio g_1/F_1 are presented at two different points Q^2. A numerical integration in (4) was performed with $f(Q^2) = -\ln Q^2$ (fit IV), corresponding to pQCD behavior. We have extrapolated the relation (20) to the region near $Q^2 = 0$. A calculation of the second part of the correction δ^P in (5) for the nonresonance region was performed by means of the Wandzura–Wilczek relation for the spin structure functions $g_1(x,Q^2)$ and $g_2(x,Q^2)$:

$$g_2(x,Q^2) = -g_1(x,Q^2) + \int_x^1 g_1(t,Q^2)\frac{\mathrm{d}t}{t}. \qquad (21)$$

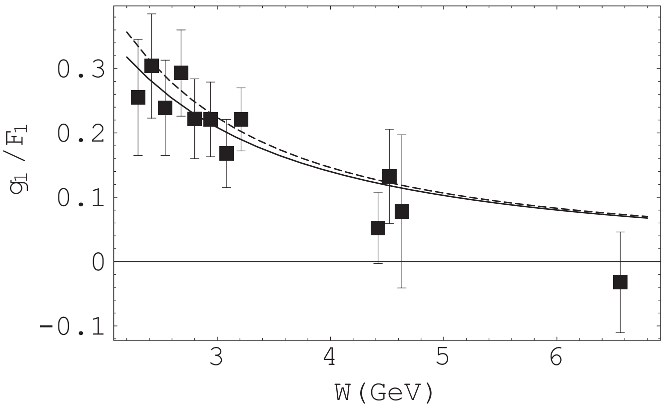

Fig. 6. $g_1(W,Q^2)/F_1(W,Q^2)$ as a function of W for the proton at $Q^2 = 0.7$ in the DIS region [12]. The dashed and solid curves are the results of fits I and IV respectively

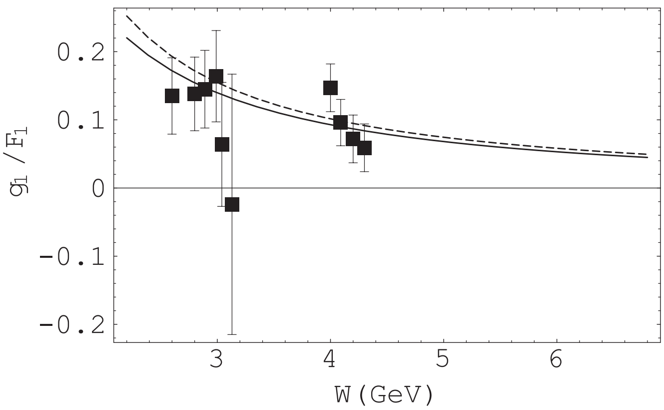

Fig. 7. $g_1(W,Q^2)/F_1(W,Q^2)$ as a function of W for proton at $Q^2 = 1.2$ in the DIS region [12]. The dashed and solid curves are the results of fits I and IV respectively

The values of the contributions δ_1^P, δ_2^P and the total contribution δ^P, obtained after the numerical integration in the resonance and nonresonance regions are as follows:

$$\delta^P_{1,\rm res} = 0.93\,\text{ppm}, \quad \delta^P_{1,\rm nonres} = 0.86\,\text{ppm},$$
$$\delta^P_1 = 1.79\,\text{ppm}, \tag{22}$$
$$\delta^P_{2,\rm res} = -0.42\,\text{ppm}, \quad \delta^P_{2,\rm nonres} = -0.01\,\text{ppm},$$
$$\delta^P_2 = -0.43\,\text{ppm}, \tag{23}$$
$$\delta^P = \delta^P_1 + \delta^P_2 = 1.4 \pm 0.6\,\text{ppm}, \tag{24}$$

where the error, indicated in expression (24), is determined by two main factors, connected with the polarized structure functions: the uncertainty of the experimental data in the nonresonance region and the possible contribution of the other baryonic resonances to the functions $g_{1,2}(\nu,Q^2)$. An estimation of the second error was done by means of the integration results in (4) and (5) for the different intervals of Q^2, W and the possible modification of the spin-dependent structure functions in the resonance region $W \geq 1.5$ GeV due to changing of the Breit–Wigner parametrization (14). The first part of the error in (24) is connected with statistical and systematical errors of the experimental data from [12].

The difference between the experimental value (1) and the theoretical result $\Delta E^{\rm th}_{\rm HFS}$ without the proton polarizability contribution can be presented in the form [2,3,37, 38]

$$\frac{\Delta E^{\rm exp}_{\rm HFS} - \Delta E^{\rm th}_{\rm HFS}}{E_F} = 4.5(1.1)\,\text{ppm}. \tag{25}$$

As was pointed out in [2,3,37], the main sources of uncertainty in this difference are the inaccuracy of the proton's form factor parameterization (dipole fit etc.) and the contradictory experimental data on the proton radius. The proton polarizability correction δ^P calculated here gives a contribution (24) of the proper sign and order of magnitude to the difference (25). Further improvement of this calculation is connected, just like the new experimental and theoretical investigations of the internal structure of the light quark baryons, and new more accurate measurements of the proton polarized structure functions, with using the QCD-based methods of the spin-dependent structure function calculation [40,41]. A more detailed consideration of the structure functions $g_{1,2}(\nu,Q^2)$ at the resonance region, taking into account contributions of some other baryonic resonances and additional decay channels is also needed. This work is in progress.

Acknowledgements. We are grateful to M.I. Eides, S.G. Karshenboim, I.B. Khriplovich, V.A. Petrun'kin, R.A. Sen'kov, for useful discussions. This work was performed under the financial support of the Russian Foundation for Fundamental Research (grant 00-02-17771) and the Program "Universities of Russia – Fundamental Researches" (grant 990192).

References

1. H. Hellwig, R.F.C. Vessot, M.W. Levine et al., IEEE Trans. IM-**19**, 200 (1970)
2. V.V. Dvoeglazov, Yu.N. Tuykhtyaev, R.N. Faustov, Phys. Part. Nucl. **25**, 58 (1994)
3. M.I. Eides, H. Grotch, V.A. Shelyuto, Theory of light hydrogenlike atoms, hep-ph/0002158 (2000)
4. C.K. Iddings, P.M. Platzman, Phys. Rev. **113**, 192 (1959)
5. S.D. Drell, J.D. Sullivan, Phys. Rev. **154**, 1477 (1967)
6. A. Verganalakis, D. Zwanziger, Nuovo Cimento A **39**, 613 (1965)
7. F. Guerin, Nuovo Cimento A **50**, 1 (1967)
8. G.M. Zinov'ev, B.V. Struminsky, R.N. Faustov, et al., Sov. J. Nucl. Phys. **11**, 715 (1970)
9. Review of Particle Physics, Eur. Phys. J. C **3**, (1998)
10. R.N. Faustov, A.P. Martynenko, V.A. Saleev, Phys. Atom. Nucl. **62**, 2099 (1999)
11. F.E. Close, An introduction to quarks and partons (Academic Press, N.Y. 1979)
12. K. Abe, et al., Preprint SLAC-PUB-7753 (1998)
13. K. Abe, et al., Preprint SLAC-PUB-7242 (1996)
14. P.L. Anthony, et al., Preprint SLAC-PUB-7983 (1999)
15. G.S. Mitchell, Preprint SLAC-PUB-8104 (1999)
16. D. Adams, et al., Preprint CERN-PPE/97-22 (1997)
17. D. Adeva, et al., Phys. Rev. D **60**, 072004 (1999)

18. V.W. Hughes, Preprint CERN-PPE/95-178, C. Aidala, A. Deshpande, V. Hughes, In Proceedings of the Workshop Polarized Protons at High Energies - Accelerator challenges and Physics Opportunities, DESY-PROC-1999-**03**, 248 (1999)
19. R.D. Erbacher, SLAC-Report-546 (1999)
20. R.L. Walker, Phys. Rev. **182**, 1729 (1969)
21. R.A. Arndt, R.L. Workman, Z. Li, et al., Phys. Rev. C **42**, 1864 (1990)
22. S. Teis, W. Cassing, M. Effenberger, et al., Z. Phys. A **356**, 421 (1997)
23. M. Effenberger, A. Hombach, S. Teis, et al., Nucl. Phys. A **613**, 353 (1997)
24. B. Krusche, J. Ahrens, G. Anton, et al., Phys. Rev. Lett. **74**, 3736 (1995)
25. N. Bianchi, V. Muccifora, E. Sanctis, et al., Phys. Rev. C **54**, 1688 (1996)
26. D. Drechsel, O. Hanstein, S.S. Kamalov, et al., Nucl. Phys. A **645**, 145 (1999)
27. Y.-B. Dong, Eur. Phys. J. A **1**, 347 (1998)
28. M. Effenberger, A. Hombach, S. Teis, et al., Nucl. Phys. A **614**, 501 (1997)
29. Z. Li, Y.-B. Dong, Phys. Rev. D **54**, 4301 (1996)
30. R. Koniuk, N. Isgur, Phys. Rev. D **21**, 1888 (1980)
31. F.E. Close, Z. Li, Phys. Rev. D **42**, 2194, 2207 (1990)
32. S. Capstick, Phys. Rev. D **46**, 1965 (1992); D **46**, 2864 (1992)
33. Zhenping Li, V. Burkert, Zhujun Li, Phys. Rev. D **46**, 70 (1992)
34. M. Warns, W. Pfeil, H. Rollnik, Phys. Rev. D **42**, 2215 (1990)
35. C.E. Carlson, N.C. Mukhopadhyay, Preprint RPI-98-N126; WM-98-105; hep-ph/9804356 (1998)
36. S.B. Gerasimov, Sov. J. Nucl. Phys. **2**, 430 (1966); S.D. Drell, A.C. Hern, Phys. Rev. Lett. **16**, 908 (1966)
37. G.T. Bodwin, D.R. Yennie, Phys. Rev. D **37**, 498 (1988)
38. S.G. Karshenboim, Phys. Lett. A **225**, 97 (1997)
39. V. Burkert, Nucl. Phys. A **623**, 59 (1997)
40. G. Altarelli, R.D. Ball, S. Forte, et al., Nucl. Phys. B **496**, 337 (1997)
41. M. Hirai, S. Kumano, M. Miyama, Comp. Phys. **108**, 38 (1998)